INVENTORY CONTROL SIMULATION WITH PROBABILISTIC DEMAND GENERATORS

FRANK T. MAYNARD

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Lieutenant Commander, Supply Corps, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
MANAGEMENT (DATA PROCESSING)

United States Naval Postgraduate School
Monterey, California

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ABSTRACT

The inventory control problem is examined. A FORTRAN program is developed, which evaluates the performance of an inventory against simulated demands derived from generators according to Poisson, negative binomial, and normal probability laws. The program is tested using data obtained from an actual inventory control point and four sets of simple decision rules. The results indicate that simulation is a good way to test decision rules and to demonstrate the results of arbitrary restraints such as budgetary restrictions.

TABLE OF CONTENTS

1. Introduction.

Not too long ago "inventory" and "wealth" were synonyms. The wealth of a medieval merchant was his inventory, the sum of all his goods and possessions. Money was one form of inventory, though a particularly useful one, since it could be exchanged readily for other forms. Interest was both illegal and immoral.

In some parts of the world, even today, a marriageable daughter displays a sizeable portion of her father's wealth by wearing the inventory of clothing and jewelry she will take with her as a dowry. The jewelry is mostly in the form of coins strung together to form necklaces or bracelets, and their number and size are a point of family pride.

In the southwestern part of the United States a rancher proudly shows his cattle to guests. It is considered poor taste, however, to enquire as to their number, for this form of inventory is a good index of his financial status.

Most American businessmen, whether private or government, have attitudes toward inventory quite different from those illustrated above. Business inventory is thought to be at best a necessary evil and at worst "the graveyard of American business" [14].

Inventory takes away the freedom to use money to meet obligations or to engage in more profitable investment. Inventory (more precisely, speculation in inventory) is blamed for excesses in business cycles. In government, inventory represents treasury borrowings and, for this reason, is considered to be subject to interest charges.

The purpose of inventory is to relieve supply from constant dependence on manufacturing or other commercial sources. If the customer is

willing to wait until the item requested can be obtained from commercial stocks or from production, there is no point in maintaining an inventory, except, of course, that buying in quantity reduces costs attributable to purchase order processing. But that is another story and beyond the scope of this paper. The purpose of inventory in the Navy is "to partially uncouple demand from commercial supply."[1] The expense of inventory is the cost of service to the customer. Given that management of the inventory is optimal, the question of inventory size boils down to a definition of the degree of service desired.

In the past the Navy has formulated budgets to maintain inventories expressed in terms of "months of supply". A month of supply is the amount of merchandise expected to be withdrawn from the inventory per month, based on historical experience and expressed in dollars. Our inventories are composed largely of insurance items.[2] Expressed in months of supply, they usually look outrageously large to the civilian leadership in the Department of Defense and in Congress, whose experience has been largely in private business.

In private business, the justification for inventory is profit. To be sure, profit in a competitive market depends, at least partly, upon customer good will. To the extent that good will demands, a commercial resale activity maintains inventory, even though that inventory may be uneconomical. Within this restraint, it works strenuously to weed out

[1] I am indebted to Cdr. S. W. Blandin, S.C., U.S.N., for this expression as well as for many of the ideas expressed in this paper.

[2] An insurance item is a slow-moving or non-moving item which is retained in inventory for service or essentiality considerations.

slow-moving items. The penalty for low or nonexistent inventory is occasional loss of sales. In the Navy, the penalty may be much higher.

The shortage penalty has been defined as the cost of a completely inoperable ship per unit of time. It is obvious, however, that not all shortages will render the ship completely inoperable. Some shortages will only partially degrade the effectiveness of the ship. Partial degradation is accounted for with the concept of essentiality; essentiality being a relative measure of the seriousness of a part shortage.

Essentiality is determined from two basic elements: (1) mission effect, and (2) compensability. "Mission effect" refers to the operational capability of the system when the part is missing, and "compensability" refers to the ability to make up for the shortage by repair, local manufacture, substitution, or cannibalization. Compensability is primarily a matter of time; that is, it determines how long the mission effect loss will be felt. Judgement of qualified personnel is used to classify items with numerical values for mission effect and compensability. Essentiality is the product of these two numerical values.

The shortage cost associated with a part shortage is the product of essentiality and the shortage penalty [11].

Shortage penalties for representative ships are as follows [11]:

Ship Type	Shortage Penalty		
	Per Year	Per Month	Per Day
Guided Missile Heavy Cruiser	$11,566,000	$963,000	$31,600
Attack Aircraft Carrier (Forrestal)	24,712,000	2,060,000	67,600
Radar Picket Destroyer	2,244,000	186,500	6,130
Radar Picket Nuclear Submarine	8,500,000	708,000	23,300
Ocean Minesweeper	690,000	57,500	1,890

The above figures are based on depreciated construction and conversion costs and total annual maintenance and operating costs.

Although the concept of essentiality has been understood for years, and although it has always been applied, on a more or less intuitive

basis, we have not yet been able to identify a significant number of the items in the Navy inventory with meaningful essentiality codes. We are, therefore, unable to describe the penalty cost of low inventory in a budget request. Essentiality is one approacn to more meaningful budgeting.

Apart from essentiality, Navy inventory managers need to justify budgets in terms of effectiveness, the ability to satisfy anticipated demands. We need to explain to Department of Defense officials and to Congress our need for inventory dollars in terms of performance, not months of supply or inventory to sales ratios. We need, most of all, some way to demonstrate to ourselves and to others the expected results of budget decisions.

In the Navy as in any large organization, we often develop complex procedures and install them without adequate testing. Such procedures frequently fail because they are based on an erroneous or partial apprehension of reality. Or, on the other hand, the procedure may be correct enough and still fail because of faulty execution. Sometimes the wherewithal to make a procedure work is missing, whether it be funds, personnel, time, computer capacity, or some other essential ingredient.

As an illustration of both kinds of failure, I offer the attempt to identify recurring and non-recurring demand. A demand which is non-recurring at a retail issue point may well be recurring when considered system-wide. For example, suppose a particular equipment is overhauled at a shipyard. As far as the shipyard is concerned, there is a one-shot requirement for repair parts. But the same equipment will be overhauled next quarter, only on a different ship at a different shipyard. Clearly

the repair parts requirements are recurring, yet it is impossible to recognize them as such at the retail level. But even more significantly, the definitions of "recurring" and "non-recurring" are so unclear and subject to various interpretation that any conclusion drawn from such categorization is wishful thinking, particularly when we remember that it is frequently personnel at the GS-3 level who make the distinction.

We are all familiar with the failure of some elaborate system which, having been installed with great fanfare, later fails decisively. We are fortunate when the failure is recognized, and the procedure is scrapped. Normally what happens is that the procedure is kept because we are "committed" or because we cannot bring ourselves to admit the mistake.

What is required is some way to evaluate decisions before we are totally committed. It is the purpose of this thesis to show how a simple simulator can help both in budgeting and in the evaluation of inventory control stockage rules.

2. Inventory Policy.[1]

The inventory control problem consists of two major questions: when to reorder and how much. In the ideal situation the demand to be experienced during lead time is known, as is the lead time. By lead time, we mean the total elapsed time between recognition of the need to reorder and actual receipt of the merchandise. The reorder point is set at precisely the known demand corresponding to the known lead time. The order quantity is set by a formula which balances ordering costs against holding costs and calculates an "economic order quantity". At the expiration of lead time an optimal quantity resupply is delivered just as the last unit of the old inventory is shipped. Customer service has been perfect, and total expense has been minimized.

Life in the real world of inventory management is not so predictable as in the utopia described above. Demand fluctuates widely and unpredictably; lead time varies; inventory restrictions are arbitrarily imposed through budget restraints. Management reacts to these realities by striving for the best balance between service and inventory investment. Policies are translated into decision rules which control the answers to the questions, when to reorder and how much.

The U. S. Navy Ships Parts Control Center attains budget objectives by adjusting two arbitrary constants and by restraining economic order quantity and risk. Let us examine SPCC's system more closely as an

[1]Much of the advanced thinking on Navy inventory management has been done at the Ships Parts Control Center, Mechanicsburg, Pennsylvania. The material in this section is drawn largely from the reports written by the Special Assistants for Advanced Logistics Research and Development, of S.P.C.C.

illustration of scientific inventory management in the real world.

The formula for economic order quantity used at SPCC is as follows:

$$EOQ = \sqrt{\frac{4Q \times C \times 2}{I \times P}} \cdot \lambda_1$$

where:

Q = quarterly demand

C = order cost

I = holding cost rate

P = unit price

λ_1 = a constant used for attaining management objectives (which, being interpreted, means living within the budget).

SPCC imposes a further control by restricting the economic order quantity to values between one quarter's demand and five years' demand.

At SPCC risk is defined as the acceptable probability of running out during lead time. Risk is calculated as follows:

$$R = \frac{PxI \times EOQ}{4Q \times S} \times \lambda_2$$

where:

PxI = holding cost

$4Q$ = annual demand

S = shortage cost

EOQ = economic order quantity

λ_2 = a multiplier

Risk is normally restricted to values between one percent and 50 percent. Protection level, which is defined as one minus risk, is used in the

computation of variable safety level.

At this stage, we must leave statistics and begin to use the techniques of probability. No matter how stable our usage rates might be - and usage rates in the Navy are almost never stable - any forecast of future usage is sure to be somewhat in error. For this reason, we base our reorder point on some type of projection of past demand <u>plus</u> a safety level to allow for variability. The amount of this variable safety level depends on the protection we desire for each line item. If we eliminate safety level and set reorder point at exactly the forecast of demand expected during lead time we run a fifty percent chance of running out if demands are normally distributed about some mean or average value.

The average demand is forecasted using exponential smoothing, a technique originated by Robert G. Brown [1]. The formula is as follows:

$$Q = aD + (1-a)\,\bar{Q}$$

where:

Q = new forecast of quarterly demand

a = smoothing weight or constant

D = demand experienced during past quarter

$\bar{Q}$ = forecast for past quarter

The beauty of exponential smoothing is that it retains the effect of past experience without our having to store large amounts of raw data. Another desirable quality is that later data are weighted more heavily than older data, the value of the weighting being a function of the smoothing constant. A further convenience lies in our ability to vary the smoothing constant to obtain either gradual or rapid response to

change, as desired. For a new item, for example, the constant can be set high, say at .3 to .5. As usage data are acquired, the constant can be lowered, to .1 perhaps, or even lower, so as to give less relative effect to last quarter's demand and more to older history.

We are interested not only in the average demand experienced in the past but also in the variations from this average. Specifically we are interested in determining whether or not past demands fit any known probability distribution. If they do, we can use this distribution to predict the probability of our being able to satisfy forecasted demand with a given stock.

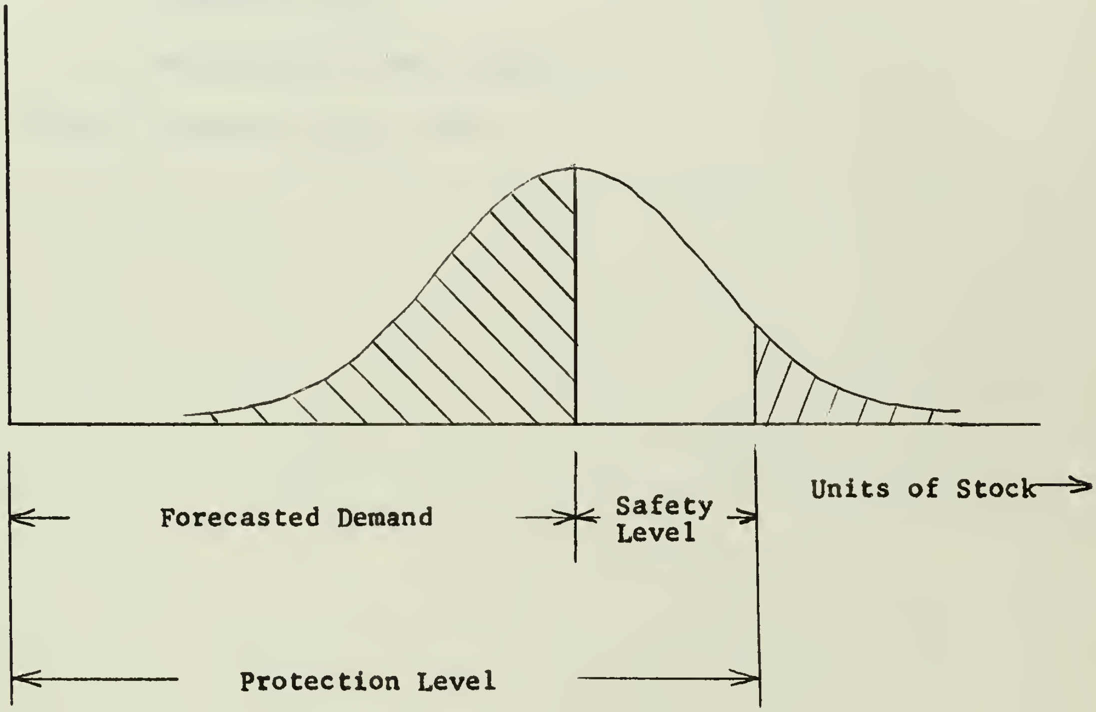
By careful analysis, SPCC has found that its items fit three distributions fairly well. If the quarterly demand is .5 or less, SPCC assumes a Poisson distribution. If the demand is greater than .5 but less than or equal to 25, the negative binomial distribution is assumed. If the demand is greater than 25, it is assumed to be normally distributed. These distributions are considered further in Appendix I.

To see how the reorder point includes both forecasted demand and variable safety level let us look at a normally distributed item.

The protection level is computed by recursively figuring the probability that demand during lead time will be less than or equal to an increasing number of units of stock. When this probability builds up to the previously calculated protection level the process is stopped and the number of units corresponding to that probability is the reorder point less program stock and obligations.

We see, therefore, that the inventory manager implements a budget restraint by scaling down the protection level through the adjustment

of the parameters which govern its calculation. The effects of such a restraint are not felt immediately, for lead times for our technical items are in the order of quarters, or even years. The effect of a lower budget for stock replenishment will begin to manifest itself toward the end of the smallest lead time and will become increasingly evident thereafter.



The forecasted demand protects the lower half of the demand distribution. The addition of variable safety level takes us up to the protection level previously calculated. The area to the right of the protection level is the risk.

Reorder point, then, is calculated as follows:

$$RP = LQ + \overline{VSL} + P + O$$

where:

L = lead time in quarters plus one standard deviation to protect against lead time variations

Q = quarterly demand forecast

P = program stock

O = obligations (back orders)

$\overline{VSL}$ = variable safety level

3. Inventory Control Simulation.

In 1962, International Business Machines Corporation completed its Modular Inventory Management Simulator, an elaborate computer program in the FORTRAN compiler language. The program is segmented so as to permit great flexibility. It is possible, for example, to use one of the forecasting routines in the program or to insert the routine used locally. The program includes all the functions normally associated with inventory management: demand forecasting, error measurement, sales, orders, receipts, etc.

The program written for this thesis is an attempt, on a much smaller scale, to show how demand can be simulated according to probability distributions and how such simulated demand can be used to evaluate inventory performance under various sets of decision rules. The demand generators are subroutines, which can be used in any program where random variable generators are required. The program itself is reproduced in Appendix II.

The first ten instructions of the main program accomplish the preliminary housekeeping tasks of recording the number of line items under consideration, setting aside storage space for the variables, zeroizing the distribution counters and starting the random and normal (0,1) number generators.

The second segment reads in the raw data, computes the probability distribution parameters for each item, identifies the distribution applicable to each item, counts the number of items applicable to each distribution and calculates for each item the average quantity per requisition. In a normal situation these data would be continuously

available as one of the requirements of normal business.

The expected demand for each item is calculated by averaging the first three quarters and then applying the exponential smoothing formula to this average and the fourth quarter demand. A high smoothing constant, 0.3, is used because the data came from a new inventory management center and its demands are not believed to have settled down yet. Similarly, for the first three quarters the squared demands are averaged. The expected squared demand is computed from this average and the fourth quarter squared demand by means of exponential smoothing with the same smoothing weight, 0.3. The variance is then obtained by the familiar formula:

$$\sigma^2 = E(x^2) - \left[E(x)\right]^2$$

In assigning items to probability distributions it is necessary to test the standard deviation-to-mean ratio of items having average quarterly demands of over 25. If this ratio exceeds three the item is identified as negative binomial distributed. This step is required because the probability of having a negative demand must be zero, or practically zero. If the mean is three times the standard deviation and the normal distribution holds, the probability of a negative demand is negligible (.0035).

The program now completes setting up the problem by recording stock quantities in accordance with decision rules which may be simple or very elaborate. These rules are in a subroutine, which can be changed at will. The inventory is priced, extended, and totaled.

Finally, the program secures simulated demands from the demand generators, extends and totals issues, and calculates effectiveness, the

ratio of total quantity issued on demand to total quantity demanded.

FORTRAN is the computer language used for this thesis. More specifically, the program was written for the FORTRAN 60 compiler for the Control Data Corporation 1604 computer.

FORTRAN permits the use of symbolic machine coding interspersed with FORTRAN statements. Symbolic coding is used in the subroutines which generate pseudo random numbers and pseudo random normal (0,1) random variables; and its use, though highly desirable, does reduce the generality of the program, for the symbolic language is that of the 1604. Other machines have different symbolic languages, and the instructions would require translation before the program could be run on another computer. One possible solution would be to substitute generators written for another machine. Such generators are widely used and should be obtainable.

A second limitation in these two subroutines is that both presume a 48-bit word length.

The Control Data 1604 general purpose digital computer at the Naval Postgraduate School was available for this project. This computer has a storage capacity of 32,768 48-bit words.

4. Demand Generators.

The simplest of the three demand generators is that for the normal distribution. A subroutine was available at the Postgraduate School which generates pseudo random normal (0,1) variables. The number secured from this subroutine is converted by multiplication by the standard deviation of the item and then adding the predicted quarterly demand. The result is rounded off to the nearest whole number.

For the Poisson and negative binomial items a pseudo random number in the range zero to one is secured from a generator, which was also available. This number is regarded as a cumulative probability mass. The probability masses for the item under consideration are calculated and summed thus: $P(0) + P(1) + P(2) + \ldots$ until the total mass equals or exceeds the random number. The last and highest value of the random variable is the simulated demand. Computationally, it is simpler to compute P(0), subtract the random number, and then continue to calculate and add probabilities to this difference until it becomes either zero or positive.

For the Poisson distribution

$$P(0) = e^{-q}$$

where:

$P(0)$ = the probability that the random variable is zero

e = the natural base, 2.718

q = the forecasted quarterly demand

Thereafter, probabilities are calculated recursively according to the formula

$$P(I) = \frac{P(I-1) \times q}{I}$$

where:

$P(I)$ = the probability that the random variable is equal to I

$P(I-1)$ = the probability that the random variable is equal to I-1

q = the forecasted quarterly demand

The iterations are terminated when I = 10, since the probability that the random variable will exceed 10 is negligible.

For the negative binomial generator the parameters p,q, and r must be computed as follows:

$$p = \frac{AM}{V}$$

$$q = 1-p$$

$$r = \frac{AM \times p}{q}$$

where:

p = the probability of success for a single Bernoulli trial

q = the probability of failure for a single Bernoulli trial

r = the number of successes (called AR in the program)

AM = the forecasted quarterly demand

V = the variance in quarterly demand

The random variable is the number of failures before the rth success. The probability of zero is thus:

$P(0) = p^r$, and thereafter

$$P(I) = \frac{P(I-1) \times (r + I-1) \times q}{I}$$

In the iterative scheme of this program the largest value that is permissible for I is 16,383, this figure being the upper limit for indexing a "do loop". This limitation could be overcome at the expense of a few more instructions.

The purpose of all the demand subroutines is to generate random "noise" around the demand forecasted by some manipulation of historical data.

5. Testing the Program.

The raw data used for the test consisted of four quarters of demand information on 277 items in Federal Supply Class 2815, Diesel Engines and Repair Parts. The data were secured from the Defense Construction Supply Center, Columbus, Ohio, and are typical of demand in repair parts classes in that high variance-to-mean ratios are prevalent. The breakdown into fast, medium, and slow movers was not typical since 94 (34%) of the items had average demands of over 25 per quarter, 155 (56%) had average quarterly demands of 25 or less but greater than or equal to 0.5, while only 28 (10%) had average quarterly demands of less than 0.5. The corresponding (estimated) percentages on all current SPCC items are, by way of contrast, 14.8%, 26.1%, and 59.1%, respectively.

It is recognized that 277 items and four quarters of usage data are not enough for statistical accuracy. They are enough, however, to show whether or not the program works.

When the data were smoothed and checked for high variance-to-mean ratios, only eight of the 277 items were identified as normally distributed, while 245 were negative binomial distributed, and 24 were Poisson distributed.

Four sets of very simple stockage rules were tested.

Forecasted Quarterly Demand	Unit Price	Stock
First Rules		
Poisson Items		
$> .25$	$> \$100$	2 Ea
$> .25$	$\leq \$100$	3 Ea
$\leq .25$	$> \$100$	1 Ea
$\leq .25$	$\leq \$100$	2 Ea
Negative Binomial and Normal Items		
All	$> \$100$	U
All	$\$100 \geq U/P > \$10$	U+S
All	$\leq \$10$	U+2S
Second Rules		
Poisson Items Same as Above		
Negative Binomial and Normal Items		
All	All	U
Third Rules		
Poisson Items Same as Above		
Negative Binomial and Normal Items		
All	$> \$50$	U
All	$\leq \$50$	U+S
Fourth Rules		
Poisson Items		
$> .25$	$< \$100$	1 Ea
Otherwise		0
Negative Binomial and Normal Items		
All	$> \$100$	.5U
All	$\$100 \geq U/P > \$50$	U
All	$\$50 \geq U/P > \$10$	U+S
All	$\leq \$10$	U+2S

Where U is the forecasted quarterly demand and S is the quarterly demand standard deviation.

For each set of stockage rules, stock levels were established. 50 simulation runs were then made using demands from the demand generators. The average results are shown below. They show how effectiveness, dollar sales, and sales-to-inventory ratios are affected by changes in stockage decision rules. The evaluation of these and any other results depends on the goals of management.

In the first three sets of rules, the 24 Poisson items were held at relatively high inventory levels. For the remaining items, various combinations of unit price criteria were used to determine the size of the variable safety level. The second set of rules is particularly interesting in this respect because it shows the result to be expected if no safety level is provided.

The fourth set of rules drastically reduced the inventory levels for Poisson items and applied more elaborate criteria to the remainder. The result was a dollar inventory of only 76 percent of the first one, which yielded comparable effectiveness and a better sales-to-inventory ratio. This result was possible because the seldom demanded Poisson items tended to be high priced.

Inventory	Sales	Sales/Inventory	Effectiveness
First Rules			
$276,615.98	$143,501.35	.518	.952
Second Rules			
$182,009.56	$118,389.19	.651	.680
Third Rules			
$239,530.38	$136,101.93	.569	.878
Fourth Rules			
$211,419.04	$118,329.89	.561	.947

6. Conclusions.

In significant positions - notably at the top - the Department of Defense is staffed with civilian personnel who understand mathematical models, gaming, and probability theory. These people rightly insist on discussing budget problems in such terms. It behooves us in the military to learn to speak their language. Inventory control simulation with probabilistic demand generation is one way to do so.

To be useful to an inventory control point, a simulator needs to be more elaborate than the one written for this thesis. It should be consistent with the other routines of the center, so that it would be possible, at any time, to leave off normal processing and project as far into the future as is desired, using simulated demand and, if desired, revised decision rules. It should take into consideration both anticipated receipts (dues) and back orders (obligations). It should establish dues and obligations and should record receipts and the release of obligations.

The IBM Modulator Inventory Simulator does all these things. Together with probabilistic demand generators, it could form the basis of a local program.

The Navy is always faced with the task, not only of understanding its problems, but of communicating them to the civilian leadership and to Congress. Inventory control simulation can help us do both. It is, first of all, a good way to look at our own business and at proposed changes. It is, further, a way to present budgets, showing clearly the anticipated results of various levels of funding. It deserves serious consideration as a management tool.

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APPENDIX I

PROBABILITY DISTRIBUTIONS

The normal probability law is too well known to require treatment in this paper. It is thoroughly discussed in almost any probability text.

Less familiar is the Poisson distribution, also called the law of rare events. It is defined as:

> The number of occurrences of events of a specified type in a period of time of length 1 when events of this type are occurring randomly at a mean rate λ per unit time [7].

The Poisson probability mass function is

$$e^{-\lambda} \frac{\lambda^x}{x!} \qquad x = 0,1,\ldots$$

Its mean and variance are both equal to λ.

Even less familiar is the negative binomial or Pascal distribution. This distribution is defined as:

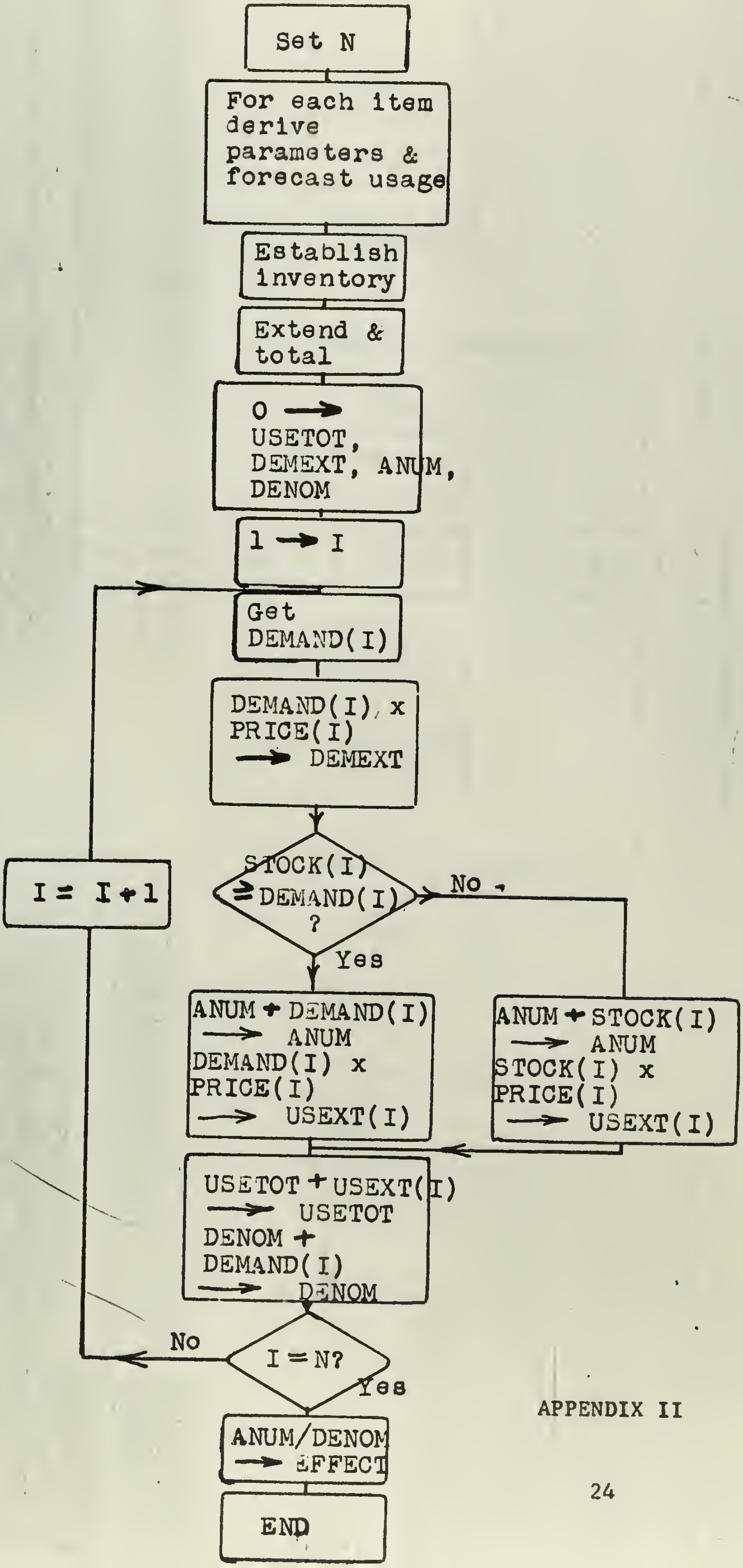
> The number of failures encountered in a sequence of independent Bernoulli trials (with probability p of success at each trial) before the rth success [7].

The negative binomial mass function is

$$\binom{r+x-1}{x} p^r q^x \qquad x = 0,1,\ldots \;. \qquad r = 1,2,\ldots \;. \qquad 0 \leq p \leq 1 \;.$$

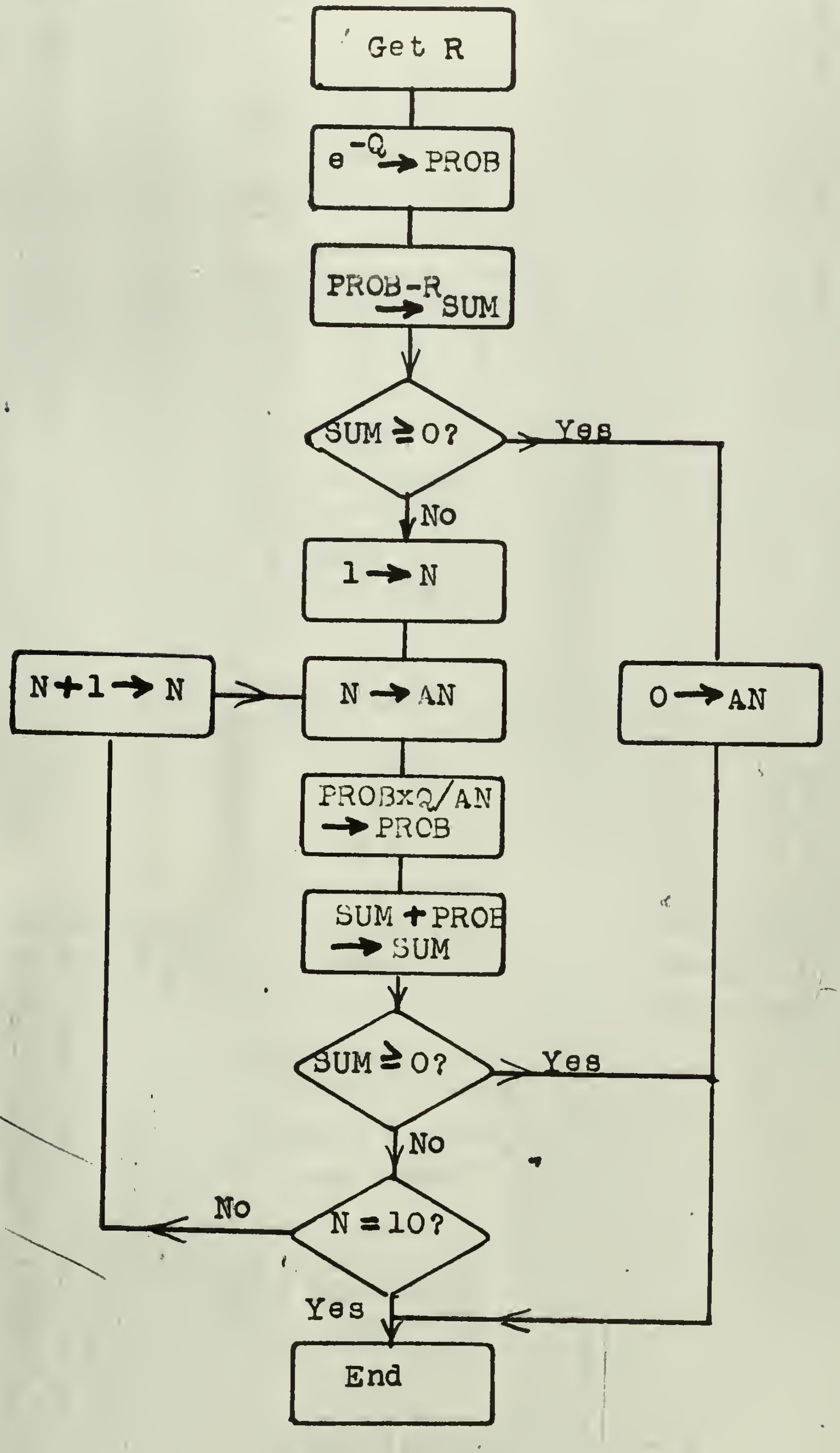
Its mean is $\frac{rq}{p}$ while the variance is $\frac{rq}{p^2}$. The negative binomial fits the demand distribution of many items of supply. One of its desirable properties is that, while the probability of the random variable being equal to zero may be quite high, it is impossible for it to be negative. This is a characteristic which makes the negative binomial law fit demand experience of items having relatively high variance to mean ratios, where the normal distribution would be inappropriate.

Main Program

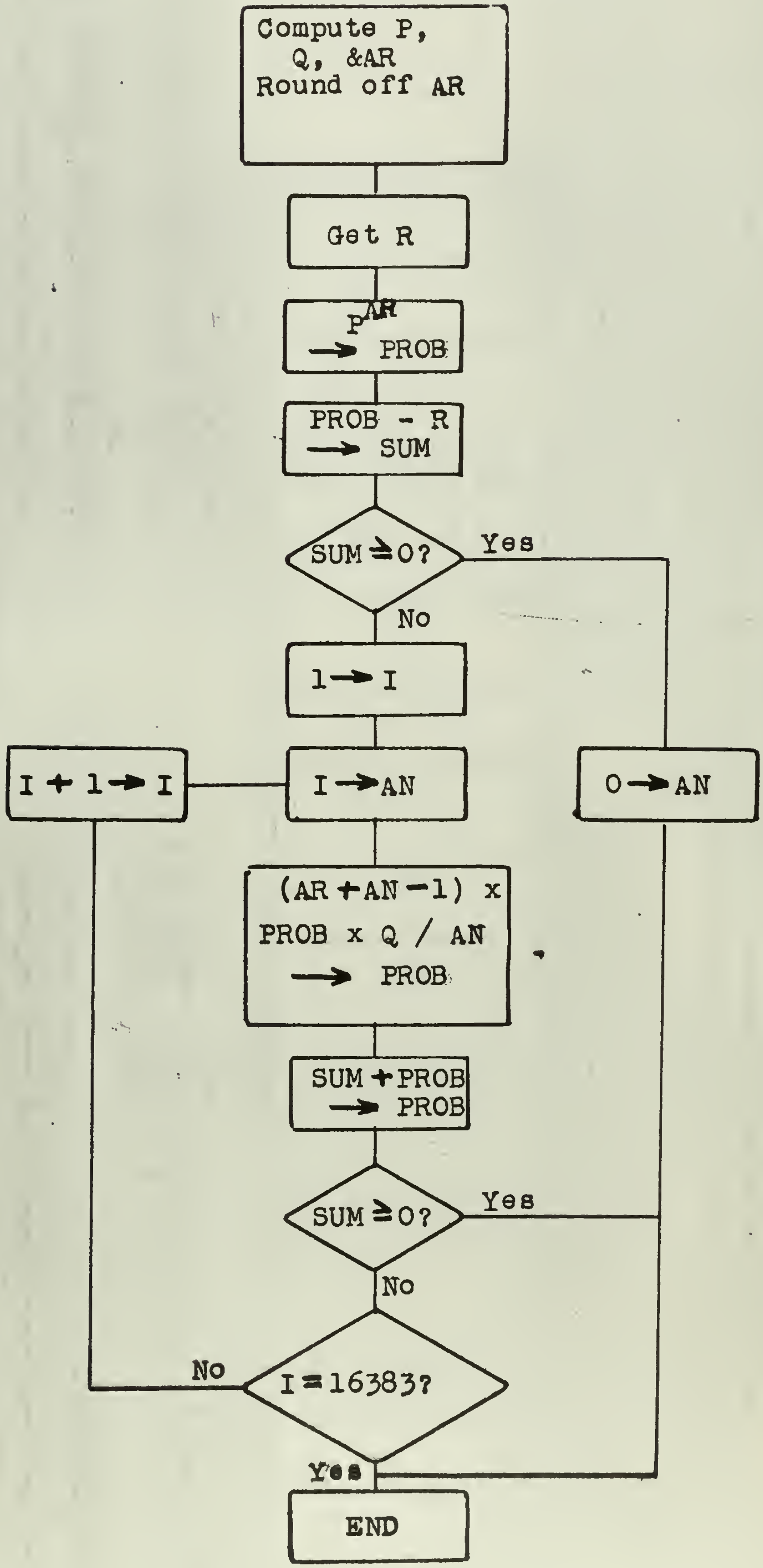


APPENDIX II

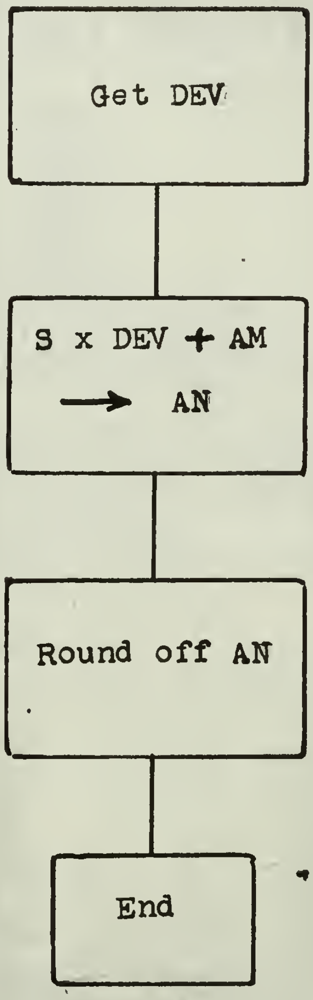
Poisson Demand Generator



Negative Binomial Demand Generator



Normal Demand Generator



```
      PROGRAM DATA1
      N=277
      DIMENSION QUSAGE(300),VAR(300),REQAVE(300),
     1ISTKNR(300),ATIME(300),SIGMA(300),
     2INDIC(300),PRICE(300),STOCK(300),
     3DEMAND(300),STKEXT(300),USEXT(300)
      COMMON QUSAGE, STOCK, INDIC, PRICE, SIGMA
      NUNIF=1695
      IBINOM=0
      NEGBIN=0
      IPOIS=0
      CALL RANDOM(0,R)
C     SETUP PECULIAR TO TEST DATA
      DO 15 I=1,N
   10 READ 2 ISTKNR(I),PRICE(I),IREC2,IQREC2,
     1NREC2,IQNREC2,IREC3,IQREC3,NREC3,IQNREC3,
     2IREC4,IQREC4,NREC4,IQNREC4,IREC1,IQREC1,
     3NREC1,IQNREC1
    2 FORMAT(I7,12X,F7.2,5X,8(I2,I4))
      NR1 = IREC1 + NREC1
      NR2 = IREC2 + NREC2
      NR3 = IREC3 + NREC3
      NR4 = IREC4 + NREC4
      QTY1=IQREC1+IQNREC1
      QTY2=IQREC2+IQNREC2
      QTY3=IQREC3+IQNREC3
      QTY4=IQREC4+IQNREC4
      NR = NR1 + NR2 + NR3 + NR4
      QTY=QTY1+QTY2+QTY3+QTY4
      QUSAGE(I)=(QTY2+QTY3+QTY4)/3.
      AVESQ=(QTY2*QTY2+QTY3*QTY3+QTY4*QTY4)/3.
      ALPHA=.3
      QUSAGE(I)=ALPHA*QTY1+(1.-ALPHA)*QUSAGE(I)
      AVESQ=ALPHA*QTY1*QTY1+(1.-ALPHA)*AVESQ
C     CALCULATION OF VARIANCE AND STANDARD DEVIATION
      VAR(I)=AVESQ-QUSAGE(I)*QUSAGE(I)
      SIGMA(I)=SQRTF(VAR(I))
C     IDENTIFICATION OF DEMAND DISTRIBUTION
      IF(SIGMA(I)-QUSAGE(I)/3.) 8,8,4
    8 IF (QUSAGE(I)-25.) 4,4,3
    3 INDIC(I)=1
      IBINOM=IBINOM+1
      GO TO 7
    4 IF(QUSAGE(I)-.5) 6,6,5
    5 INDIC(I)=2
      NEGBIN=NEGBIN+1
      GO TO 7
    6 INDIC(I)=3
      IPOIS=IPOIS+1
    7 ANR = NR
   15 REQAVE(I)=QTY/ANR
      STKSUM=0.
      CALL BUY1(N)
C     EXTEND INVENTORY AND TOTAL
      DO41 I=1,N
      STKEXT(I)=PRICE(I)*STOCK(I)
   41 STKSUM=STKSUM+STKEXT(I)
```

```
C     GET DEMAND FROM DEMAND GENERATORS AND EXTEND
      USETOT=0.
      DEMEXT=0.
      ANUM=0.
      DENOM=0.
      DO48 I=1,N
      IF(INCIC(I)-2) 42,43,44
   42 CALL NORMAL (NUNIF,QUSAGE(I),SIGMA(I),AN)
      GO TO 45
   43 CALL NEGBINO(QUSAGE(I),VAR(I),AN)
      GO TO 45
   44 CALL POISSON(QUSAGE(I), AN)
   45 DEMAND(I)= AN
      DEMEXT=DEMEXT+DEMAND(I)*PRICE(I)
C     CALCULATE EFFECTIVENESS
      IF(STOCK(I)-DEMAND(I)) 49,46,46
   49 ANUM=ANUM+STOCK(I)
      USEXT(I)=STOCK(I)*PRICE(I)
      GO TO 47
   46 ANUM=ANUM+DEMAND(I)
      USEXT(I)=DEMAND(I)*PRICE(I)
   47 USETOT=USETOT+USEXT(I)
   48 DENOM=DENOM+DEMAND(I)
      EFFECT=ANUM/DENOM
   52 PRINT 51 STKSUM,USETOT,EFFECT
   51 FORMAT(1X,3(F15.3))
      END
```

```
C        POISSON DEMAND GENERATOR
         SUBROUTINE POISSON(Q,AN)
      1  CALL RANDOM(1,R)
      2  PROB=2.718**(-Q)
      3  SUM=PROB-R
      4  IF(SUM) 7,5,5
      5  AN=0.
      6   GO TO 13
      7  DO 12 N=1,10
      8  AN=N
      9  PROB=PROB*Q/AN
     10  SUM=SUM+PROB
     11  IF(SUM)12,13,13
     12  CONTINUE
     13  END
C
C        NEGATIVE BINOMIAL DEMAND GENERATOR
         SUBROUTINE NEGBINO (AM, V, AN)
      1  P = AM / V
C        P RESTRICTED TO VALUES BETWEEN .000001 AND .999999
         IF(P-.000001) 202,2,200
    200  IF(P-.999999) 2,2,201
    201  P=.999999
         GO TO 2
    202  P=.000001
      2  Q = 1.-P
      3  AR=AM*P/Q+.5
      5  IAR=AR
      6  AR=IAR
      7  IF(AR) 8,8,10
      8  AR=1.
     10  CALL RANDOM (1,R)
     11  PROB = P**AR
     12  SUM = PROB - R
     13  IF(SUM) 16,14,14
     14  AN=0.
     15  GO TO 22
     16  DO 21 I=1,16383
     17  AN=I
     18  PROB=(AR+AN-1.)/AN*PROB*Q
     19  SUM = SUM + PROB
     20  IF(SUM) 21,22,22
     21  CONTINUE
     22  END
C
C        NORMAL DEMAND GENERATOR
         SUBROUTINE NORMAL (NUNIF,AM,S,AN)
      1  CALL RNDEV (NUNIF,DEV)
      3  AN=S*DEV+AM+.5
         N=AN
         AN=N
      4  END
C
```

```
C       NORMAL (C,1) RANDOM VARIABLE GENERATOR
        SUBROUTINE RNDEV(NUNIF,DEV)
        CON(NZ=77777777777777778).
        RSV(ERA=1).
        ENA(0)          STA(ERA)
        SIL1(8)         ENI1(15)
  8RDV  LDA(NUNIF)      MUI(1220703125)
        SCL(7600000000000000B)  STA(NUNIF)
        RAD(ERA)        IJP1(8RDV)
        SUB(2000000000000000B)  STA(ERA)
        AJP2(L+1)       LAC(ERA)
        ENA1(0)         LLS(36)
        SCA1(2057B)     LRS(47)
        SSK(ERA)        SCM(NZ)
        SCM(NZ)         FMU(1724673317272054B)
      8 STA(DEV)        ENI1(N).
        RETURN
        END
C
C       RANDOM NUMBER GENERATOR
        SUBROUTINE RANDOM (K,R)
      1 IF(K) 5,2,5
B     2 E= 4000 0000 0000 0000
B     3 E2=2000 0000 0000 0000
B     4 X = 1777 7777 7777 7777
      5 SIU6(L+7),ENI6(2),ENQ(N),LDA(X),LLS(19),
        SCL(E),ADD(X),SCL(E),IJP6(L-1),STA(X),
        ARS(11),ADD(E2),FAD(E2),STA(R),ENI6(N),
        ENI(N)
      6 END
```

```
C         FIRST SET OF STOCKAGE RULES
          SUBROUTINE BUY1(N)
       10DIMENSION QUSAGE(300),STOCK(300),
        1INDIC(300),PRICE(300),SIGMA(300)
          COMMON QUSAGE,STOCK,INDIC,PRICE,SIGMA
          DO20 I=1,N
C         IDENTIFICATION OF POISSON DISTRIBUTED ITEMS
          IF(INDIC(I)-2) 10,10,2
C         POISSON RULES
        2 IF(QUSAGE(I)-.25) 6,6,3
        3 IF(PRICE(I)-100.) 5,5,4
        4 STOCK(I)=2.
          GO TO 20
        5 STOCK(I)=3.
          GO TO 20
        6 IF(PRICE(I)-100.) 8,8,7
        7 STOCK(I)=1.
          GO TO 20
        8 STOCK(I)=2.
          GO TO 20
C         NORMAL AND BINOMIAL RULES
       10 IF(PRICE(I)-100.) 12,12,11
       11 ISTOCK=QUSAGE(I)+.5
          GO TO 19
       12 IF(PRICE(I)-10.) 14,14,13
       13 ISTOCK=QUSAGE(I)+SIGMA(I)+.5
          GO TO 19
       14 ISTOCK=QUSAGE(I)+2.*SIGMA(I)+.5
C         ROUND OFF TO NEAREST WHOLE NUMBER
       19 STOCK(I)=ISTOCK
       20 CONTINUE
       25 END
C
C         SECOND SET OF STOCKAGE RULES
          SUBROUTINE BUY1(N)
       10DIMENSION QUSAGE(300),STOCK(300),
        1INDIC(300),PRICE(300),SIGMA(300)
          COMMON QUSAGE,STOCK,INDIC,PRICE,SIGMA
          DO20 I=1,N
C         IDENTIFICATION OF POISSON DISTRIBUTED ITEMS
          IF(INDIC(I)-2) 10,10,2
C         POISSON RULES
        2 IF(QUSAGE(I)-.25) 6,6,3
        3 IF(PRICE(I)-100.) 5,5,4
        4 STOCK(I)=2.
          GO TO 20
        5 STOCK(I)=3.
          GO TO 20
        6 IF(PRICE(I)-100.) 8,8,7
        7 STOCK(I)=1.
          GO TO 20
        8 STOCK(I)=2.
          GO TO 20
C         NORMAL AND BINOMIAL RULES
       10 ISTOCK=QUSAGE(I)+.5
C         ROUND OFF TO NEAREST WHOLE NUMBER
       17 STOCK(I) = ISTOCK
       20 CONTINUE
       25 END
C
```

```
C      THIRD SET OF STOCKAGE RULES
       SUBROUTINE BUY1(N)
     10DIMENSION QUSAGE(300),STOCK(300),
      1INDIC(300),PRICE(300),SIGMA(300)
       COMMON QUSAGE,STOCK,INDIC,PRICE,SIGMA
       DO20 I=1,N
C      IDENTIFICATION OF POISSON DISTRIBUTED ITEMS
       IF(INDIC(I)-2) 10,10,2
C      POISSON RULES
     2 IF(QUSAGE(I)-.25) 6,6,3
     3 IF(PRICE(I)-100.) 5,5,4
     4 STOCK(I)=2.
       GO TO 20
     5 STOCK(I)=3.
       GO TO 20
     6 IF(PRICE(I)-100.) 8,8,7
     7 STOCK(I)=1.
       GO TO 20
     8 STOCK(I)=2.
       GO TO 20
C      NORMAL AND BINOMIAL RULES
    10 IF(PRICE(I)-50.) 11,11,12
    11 ISTOCK=QUSAGE(I)+SIGMA(I)+.5
       GO TO 17
    12 ISTOCK=QUSAGE(I)+.5
C      ROUND OFF TO NEAREST WHOLE NUMBER
    17 STOCK(I) = ISTOCK
    20 CONTINUE
    25 END
C
C      FOURTH SET OF STOCKAGE RULES
       SUBROUTINE BUY1(N)
     10DIMENSION QUSAGE(300),STOCK(300),
      1INDIC(300),PRICE(300),SIGMA(300)
       COMMON QUSAGE,STOCK,INDIC,PRICE,SIGMA
       DO20 I=1,N
C      IDENTIFICATION OF POISSON DISTRIBUTED ITEMS
       IF(INDIC(I)-2) 10,10,2
C      POISSON RULES
       IF(PRICE(I)-100.) 2,6,6
     2 IF(QUSAGE(I)-.25) 6,6,3
     3 STOCK(I)=1.
       GO TO 20
     6 STOCK(I)=0.
       GO TO 20
C      NORMAL AND BINOMIAL RULES
    10 IF(PRICE(I)-100.) 12,12,11
    11 ISTOCK=.5*QUSAGE(I)+.5
       GO TO 19
    12 IF(PRICE(I)-50.) 14,14,13
    13 ISTOCK=QUSAGE(I)+.5
       GO TO 19
    14 IF(PRICE(I)-10.) 16,16,15
    15 ISTOCK=QUSAGE(I)+SIGMA(I)+.5
       GO TO 19
    16 ISTOCK=QUSAGE(I)+2.*SIGMA(I)+.5
    19 STOCK(I)=ISTOCK
    20 CONTINUE
    25 END
```